Theory of hitches

Benjamin F. Bayman
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
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If one end of a rope is tied to a pole and the other end is pulled, the hitch may slip or it may hold fast. In this paper, we present a method of predicting which will happen. The topology of the hitch determines an inequality involving the coefficients of friction characteristic of the rope and pole. If this inequality is satisfied, the different turns of the hitch press on each other, and on the pole, to produce a self-locking unit that can withstand an arbitrarily strong pull.

I. INTRODUCTION

The introductory physics course probably exceeds all other courses in the extent to which it gives a student an understanding, in terms of a few basic principles, of phenomena he encounters in everyday life. These points of contact between physical theory and everyday experience are of great pedagogical value. They emphasize that physics is concerned with the real world, and they illustrate the abstractions we have to make to express a complex situation in terms amenable to a simple analysis. We present here a somewhat unusual example of the application of physics to everyday life: the role of friction and feedback in producing the remarkable holding power of knots and hitches.

We confine our attention in this paper to hitches, by which we mean knots in which a rope is tied to a pole or other cylindrical object. One end of the rope is tucked under one or more of the turns the rope makes around the pole. The other end is subjected to a pull, and a successful hitch will support a strong pull without slipping. This ability to withstand a strong pull relies upon the friction involved when rope rubs against rope and rope rubs against pole. It also depends upon the topology of the hitch. Anyone who has attempted to tie a nylon fishing line to a steel hook knows that some hitches are secure, and others are not. The object of this paper is to obtain a criterion involving friction and configuration which will indicate whether or not a hitch will hold.

Sections II and III of this paper are suitable for an introductory course. They illustrate the cooperative effect of the different segments of a hitch in forming a self-locking unit. One interesting aspect of this analysis is that it deals almost exclusively with inequalities. Also suitable for an introductory course are the general conclusions presented at the end of Sec. V, concerning the topological features possessed by the best hitches. The analysis of the general hitch given in Sec. IV uses matrix methods that are more appropriate for an intermediate-level mechanics course.

II. FRICTION

Let μ be the coefficient of static friction between the rope and the pole. Then the pole can exert a tangential force on the rope, so that T_2 in Fig. 1 can exceed T_1 (and vice versa). If $T_2 > T_1$, the rope will not slip as long as the inequality

$$T_2 \leq T_1 \exp(\mu\theta) \qquad (T_2 > T_1) \tag{1}$$

is obeyed.[1] Here θ is the angle (in radians) subtended at the axis of the pole by the arc along which the rope and pole are in contact.

In the following applications of (1), θ will always be an integral multiple of 2π. It will simplify our notation if we rewrite (1) as

$$T_2 \leq T_1 \epsilon^n \qquad (n \text{ turns}), \tag{1$'$}$$

where

$$\epsilon \equiv \exp(2\pi\mu). \tag{1$''$}$$

Figure 2 shows a situation in which a segment of rope is squeezed against the pole by another segment passing over it. The friction involved here allows the tension in the lower segment to differ on the two sides of the crossing. The limiting value of this difference is proportional to the force on the lower segment perpendicular to the surface of the pole, which is in turn proportional to the tension in the upper segment

$$T_2 \leq T_1 + \eta T \qquad (T_2 > T_1). \tag{2}$$

The constant η in (2) depends upon coefficients of friction, and upon the ratio of the diameters of the rope and pole. Note that we have assumed that the tension in the upper segment does not change at the crossing. This would be approximately the case if the friction between the ropes was much less than the friction between the rope and the pole. In Sec. VI we will discuss the consequences of relaxing this assumption. Measurements with braided nylon string on a smooth steel rod yielded $\eta \approx 0.2$ and $\epsilon \approx 4$.

III. A SIMPLE EXAMPLE: THE CLOVE HITCH

Figure 3 shows a clove hitch. Suppose that the tensions increase as we follow the rope around the hitch, so that

$$t_0 \leq t_1 \leq t_2 \leq t_3 \leq t_4.$$

Application of (1′) and (2) yields the following conditions that must be obeyed if the hitch is not to slip:

$$t_1 \leq t_0 + \eta t_2, \tag{3a}$$

$$t_2 \leq \epsilon t_1, \tag{3b}$$

$$t_3 \leq \epsilon t_2, \tag{3c}$$

$$t_4 \leq t_3 + \eta t_2 \leq (\epsilon + \eta) t_2. \tag{3d}$$

By combining (3a) and (3b) we get

$$t_2 \leq \epsilon(t_0 + \eta t_2),$$

$$t_2(1 - \eta\epsilon) \leq \epsilon t_0. \tag{4}$$

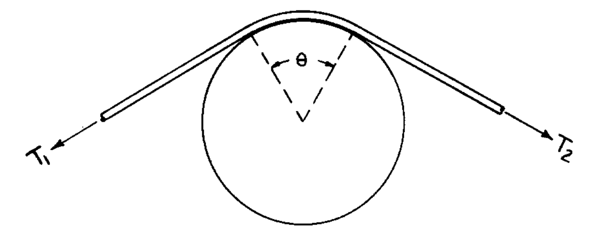


Fig. 1. Friction between rope and pole allows tensions T_1 and T_2 to differ.

We must now distinguish between two situations.

(a) Low friction:

$$\eta\epsilon < 1. \tag{5a}$$

In this case the quantity in parentheses on the left-hand side of (4) is positive, so that (4) and (3d) can be rewritten as

$$t_2 \leq [\epsilon/(1-\eta\epsilon)]t_0,$$
$$t_4 \leq [\epsilon(\epsilon+\eta)/(1-\eta\epsilon)]t_0. \tag{6}$$

Thus the hitch will not slip, provided that t_4 does not exceed t_0 by a factor greater than that given by (6).

(b) High friction:

$$\eta\epsilon > 1. \tag{5b}$$

In this case the quantity in parentheses on the left-hand side of (4) is negative, and (4) is valid for *any* non-negative tensions t_0 and t_2. It is valid even if $t_0 = 0$, and for arbitrarily large t_2. Since t_2 can become arbitrarily large, so can t_4. Thus in this high-friction situation, the clove hitch of Fig. 3 accomplishes what we require from a hitch: it will withstand an arbitrarily strong pull on one end while the other end hangs loose. In this situation we say that the hitch "holds fast."

Our aim is to find, for a hitch of any given topology, the limiting condition analogous to (5b). We will see that many hitches are superior to the clove hitch, in the sense that they will hold fast with less friction (smaller η,μ).

IV. THE GENERAL HITCH

The methods used in the previous example can be generalized as follows:

(a) Let us divide the hitch into segments. We start at the free end. The first segment starts where the free end of the rope passes under one of the turns. We follow along the rope until it again passes under a turn. This marks the end of the first segment, and the beginning of the second segment. We continue in this way along the hitch, each new segment starting where the rope passes under one of the

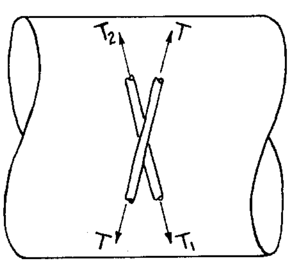


Fig. 2. Upper segment, with tension T, squeezes the lower segment against the pole.

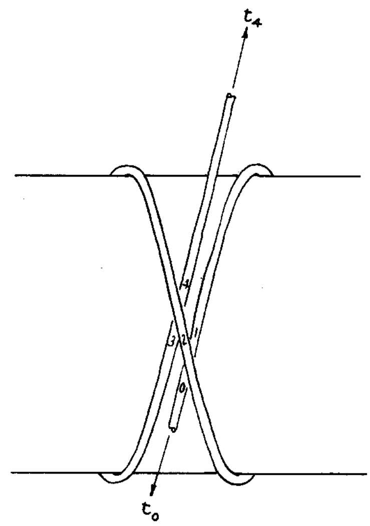


Fig. 3. Clove hitch.

turns of the hitch. We will use T_i to be the tension in the rope at the *beginning* of the ith segment. The last segment, the one that leaves the hitch at the high-tension side, will be called segment number q.

(b) Let n_i be the number of turns around the pole made by the ith segment. Since T_i is the tension at the beginning of the ith segment, $\epsilon^{n_i}T_i$ is the tension at the end of the ith segment.

(c) Let b_i be the number of the segment under which the ith segment begins.

(d) Let m_i be the number of turns from the start of the b_i segment to the place where it passes over the ith segment. Since the tension at the beginning of the b_i segment is T_{b_i}, the tension where it passes over the ith segment is $\epsilon^{m_i}T_{b_i}$. Thus (2) requires that

$$T_i \leq \epsilon^{n_{i-1}}T_{i-1} + \eta\epsilon^{m_i}T_{b_i} \qquad (T_{i-1} \leq T_i) \tag{7}$$

for $i = 2, 3, \ldots, q$ (see Fig. 4). If T_0 is the tension in the rope just before the first segment (what we have called the free end of the rope), then (7) can be supplemented by

$$T_1 \leq T_0 + \eta\epsilon^{m_1}T_{b_1} \qquad (T_0 \leq T_1). \tag{7'}$$

It is convenient to rewrite (7) and (7′) as

$$\sum_{j=1}^{q} A_{ij}T_j \leq \delta_{i,1}T_0 \qquad (T_{i-1} \leq T_i), \tag{8}$$

with the matrix A defined by

$$A_{ij} \equiv B_{ij} - \eta C_{ij}, \tag{9a}$$
$$B_{ij} \equiv \delta_{ij} - \epsilon^{n_j}\delta_{i-1,j}, \tag{9b}$$

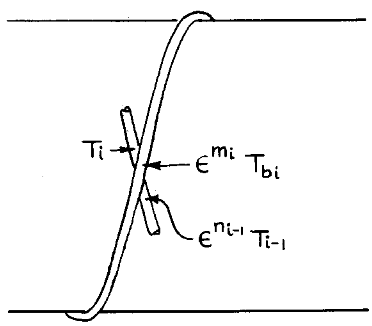


Fig. 4. Conditions at the beginning of the ith segment.

$$C_{ij} \equiv \epsilon^{m_i} \delta_{b_i,j}. \quad (9c)$$

When $\eta = 0$, $A = B$ and $\det A = 1$. Let η_c be such that

$$\det A > 0 \qquad \text{if } 0 \le \eta < \eta_c, \quad (10a)$$

$$\det A = 0 \qquad \text{if } \eta = \eta_c. \quad (10b)$$

Thus η_c depends upon the configuration of the hitch, and on the value of the friction parameter ϵ defined in (1″). In the η region defined by (10a), the reciprocal of A exists. We denote it by M:

$$MA = M(B - \eta C) = (B - \eta C)M = 1. \quad (11)$$

B and C defined in (9b) and (9c) are independent of η, but M depends upon η. It is easy to verify that when $\eta = 0$, M is given by

$$M_{ij}(0) = 0 \qquad \text{if } j > i, \quad (12a)$$

$$M_{ij}(0) = 1 \qquad \text{if } j = i, \quad (12b)$$

$$M_{ij}(0) = \epsilon^{n_j + n_{j+1} + \cdots + n_{i-1}} \qquad \text{if } j < i. \quad (12c)$$

If we differentiate (11) with respect to η, we find that

$$\frac{dM}{d\eta}(B - \eta C) - MC = 0,$$

$$\frac{dM}{d\eta} = MCM. \quad (13)$$

Since all matrix elements of $M(0)$ and C are non-negative, $dM(0)/d\eta$ given by (13) is non-negative. Thus as η increases from zero, none of the matrix elements of M will decrease from their $\eta = 0$ values given by (12). Since (13) continues to apply as long as M exists, we conclude that

$$\left.\begin{array}{ll} M_{ij}(\eta) \geqslant 0 & \text{if } j > i \\ M_{ij}(\eta) \geqslant 1 & \text{if } j = i \\ M_{ij}(\eta) \geqslant \epsilon^{n_j + n_{j+1} + \cdots + n_{i-1}} & \text{if } j < i \end{array}\right\} \; 0 \leqslant \eta < \eta_c. \quad (14)$$

Furthermore, it follows from (12) that

$$M_{i+1,j}(0) - M_{ij}(0) \geqslant 0,$$

$$M_{i,j-1}(0) - M_{ij}(0) \geqslant 0,$$

and from (13) that

$$\frac{d}{d\eta}(M_{i+1,j} - M_{ij}) = \sum_{k=1}^{n} (M_{i+1,k} - M_{ik})(CM)_{kj},$$

$$\frac{d}{d\eta}(M_{i,j-1} - M_{ij}) = \sum_{k=1}^{n} (MC)_{ik}(M_{k,j-1} - M_{ki}).$$

Arguments similar to the one used to derive (14) then show that

$$\left.\begin{array}{l} M_{i+1,j}(\eta) \geqslant M_{ij}(\eta) \quad (15a) \\ M_{i,j-1}(\eta) \geqslant M_{ij}(\eta) \quad (15b) \end{array}\right\} \; 0 \leqslant \eta < \eta_c.$$

Thus no matrix element of M is larger than M_{q1}.

As long as $0 \leqslant \eta < \eta_c$ and all matrix elements of M are non-negative, we can multiply (8) by them without changing the directions of the inequalities:

$$\sum_{k=1}^{q} M_{ki} \sum_{j=1}^{q} A_{ij} T_j \leqslant T_0 \sum_{k=1}^{q} M_{ki} \delta_{i,1} = T_0 M_{k1}.$$

Since M is the reciprocal of A, this yields

$$T_k \leqslant T_0 M_{k1} \qquad (0 \leqslant \eta < \eta_c). \quad (16)$$

Furthermore, the conditions (14) and (15a) guarantee that these T_k satisfy

$$T_k \geqslant 0, \quad (17a)$$

$$T_{k+1} \geqslant T_k, \quad (17b)$$

so that (16) provides a physically acceptable set of tensions.

The region $0 \leqslant \eta < \eta_c$ thus corresponds to the low-friction situation in (5a), in which a hitch will not slip as long as the tensions in its segments do not exceed certain multiples of T_0. In particular, if there is no tension in the free end ($T_0 = 0$), the only way in which (16) and (17a) can be simultaneously satisfied is for all the T_k to vanish, and the hitch will sustain no tension. Thus the hold-fast condition cannot be satisfied if η is less than η_c. We will now show that if $\eta \geqslant \eta_c$, it is possible to find a nonzero set T_k which satisfies the stability conditions (8), even when $T_0 = 0$.

Let $a_{ij}(\eta)$ be the *cofactor* of $A_{ij}(\eta)$, so that

$$\sum_{j=1}^{q} A_{ij} a_{kj} = \sum_{j=1}^{q} A_{ij} (\tilde{a})_{jk} = \delta_{ik} \det A. \quad (18)$$

From this it follows that

$$M_{kj}(\eta) = a_{jk}(\eta)/\det A, \quad (19)$$

and since $\det A > 0$ when $0 \leqslant \eta < \eta_c$, (14) and (15) imply that

$$\left.\begin{array}{ll} a_{jk}(\eta) \geqslant 0 & (20a) \\ a_{j,k+1}(\eta) \geqslant a_{jk}(\eta) & (20b) \\ a_{j-1,k}(\eta) \geqslant a_{jk}(\eta) & (20c) \end{array}\right\} \; 0 \leqslant \eta < \eta_c.$$

In particular, no $a_{jk}(\eta)$ is larger than $a_{1q}(\eta)$. When $\eta = \eta_c$, $\det A = 0$. Then if we define a set T_j by

$$T_j \equiv a_{1j}(\eta_c), \quad (21)$$

Eq. (18) can be written

$$\sum_{j=1}^{q} A_{ij}(\eta_c) T_j = 0, \quad (22)$$

which is equivalent to (8) with $T_0 = 0$. Thus if not all the $a_{ij}(\eta)$ vanish, the T_j defined in (21) are a nontrivial set of tensions satisfying the stability conditions for the hitch, with no tension in the free end. And these T_j, which satisfy the $T_0 = 0$ version of (8) when $\eta = \eta_c$, continue to satisfy (8) when η exceeds η_c. This is more easily seen from Eqs. (7) which are equivalent to (8). Thus apart from our proviso that not all the $a_{ij}(\eta)$ vanish, we have shown that the hold-fast condition can be satisfied when, and only when, $\eta \geqslant \eta_c$.

Finally we consider the possibility that all the $a_{ij}(\eta_c)$ vanish. All the $a_{ij}(\eta)$, and $\det A$, are polynomials in η. If η_c is a zero of $a_{1q}(\eta)$ then $a_{1q}(\eta)$ has the form

$$a_{1q}(\eta) = (\eta - \eta_c)^r \alpha(\eta), \quad (23a)$$

with

$$\alpha(\eta_c) \neq 0, \qquad r \geqslant 1. \quad (23b)$$

Since no $a_{ij}(\eta)$ exceeds $a_{1q}(\eta)$ whenever $0 \leqslant \eta < \eta_c$, it follows that if the other $a_{ij}(\eta)$ are written in the form (23), the corresponding power of $(\eta - \eta_c)$ must be at least as large as r. Since the dimension of a is q, it follows that

$$\det a = (\eta - \eta_c)^s \beta, \qquad (24a)$$

with

$$s \geqslant qr. \qquad (24b)$$

But if we take the determinant of (18), we find that

$$\det a = (\det A)^{q-1}. \qquad (25)$$

Comparison of (24) and (25) shows that $\det A$ vanishes at $\eta = \eta_c$ with a power of $(\eta - \eta_c)$ *greater* than r. Thus if $T_j(\eta)$ is defined by

$$T_j(\eta) \equiv a_{ij}(\eta)/(\eta - \eta_c)^r, \qquad (21')$$

we know that $T_q(\eta)$ approaches a finite positive value $\alpha(\eta_c)$ when $\eta \to \eta_c$, $T_{i(<q)}(\eta)$ approaches zero, or a positive value when $\eta \to \eta_c$, $\det A/(\eta - \eta_c)^r$ approaches zero when $\eta \to \eta_c$, and so (18) divided by $(\eta - \eta_c)^r$ yields

$$\sum_{j=1}^{q} A_{ij}(\eta_c)\, T_j(\eta_c) = 0 \qquad [T_j(\eta_c) \leqslant T_{j+1}(\eta_c)]. \qquad (22')$$

We see then, in all cases, when $\eta \geqslant \eta_c$ it is possible to find a nontrivial set of tensions which satisfy the stability conditions with $T_0 = 0$, whereas when $\eta < \eta_c$ these conditions can only be satisfied by the trivial solution $T_i = 0$. The hold-fast condition, generalizing the condition $\eta \geqslant 1/\epsilon$ for the clove hitch, is $\eta \geqslant \eta_c$, with η_c the smallest value of η for which $\det A$ vanishes.

The dimension of the matrix A is equal to q, the number of segments. Since no segment ever begins under the last segment, the one that leaves the hitch, the last column of A will consist of a column of $q - 1$ zeroes over a diagonal element of 1. Thus $\det A$ is unchanged if we strike off this last column and row. Alternatively, when we calculate the matrix A we need only include the first $q - 1$ rows and columns corresponding to the first $q - 1$ segments. In fact it is possible to eliminate from the matrix A every row and column referring to a segment that passes over no other segment. This leads to an equivalent condition to determine η_c:

$$\det E(\eta_c) = 0,$$

with

$$E_{ij}(\eta) \equiv \delta_{ij} - \eta \sum_{k=1}^{i} \epsilon^{m_k + n_k + n_{k+1} + \cdots + n_{i-1}}\, \delta_{b_k,j}.$$

The dimension of E is equal to the number of segments that pass *over* other segments, which is usually smaller than the dimension of A.

V. EXAMPLES

We first apply the general method to the clove hitch illustrated in Fig. 3. The first segment starts at the location of the small numeral 1 in Fig. 3. This segment encircles the pole twice, until it passes under itself just after the location of the small numeral 3. The second segment starts here and leaves the hitch. Since we can ignore the segment that leaves the hitch, our matrix A is 1×1. Segment 1 starts under segment 1, so $b_1 = 1$. Segment b_1 ($\equiv 1$) passes over segment 1 one turn beyond the beginning of segment b_1, so $m_1 = 1$. Thus (9) yields

$$A_{11} = \delta_{1,1} - \eta\epsilon^1 = 1 - \eta\epsilon = \det A$$

i	n_i	b_i	m_i
1	1	2	1
2	1	1	0

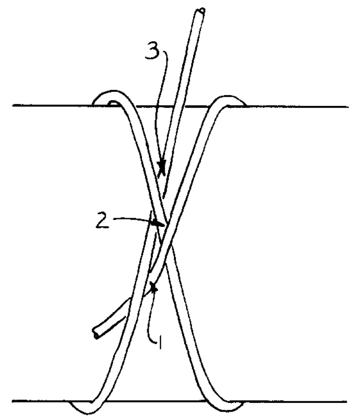


Fig. 5. Ground-line hitch. The arrows indicate the beginnings of the three segments.

and η_c, the smallest zero of $\det A$, is $1/\epsilon$. This confirms our result in Sec. III.

Closely related to the clove hitch is the ground-line hitch,[2] shown in Fig. 5. The values of n_i, b_i, and m_i, calculated as described in Sec. IV, are also shown. The matrix A is

$$A = \begin{bmatrix} 1 & -\eta\epsilon^1 \\ -\epsilon^1 - \eta\epsilon^0 & 1 \end{bmatrix}, \qquad (26a)$$

and so

$$\det A = 1 - \eta\epsilon(\eta + \epsilon). \qquad (26b)$$

The ground-line hitch holds fast when $\eta\epsilon(\eta + \epsilon) \geqslant 1$. Since ϵ [$\equiv \exp(2\pi\mu)$] always exceeds 1, it is clear that there is a range of η,

$$[\epsilon(\eta + \epsilon)]^{-1} < \eta < \epsilon^{-1} \qquad (27)$$

for which the ground-line hitch will hold fast, whereas the clove hitch will slip. The braided nylon string on a steel rod referred to in Sec. II ($\eta = 0.2$, $\epsilon = 4$) is in this range, and experiment confirms the predicted superiority of the ground-line hitch over the clove hitch.

Figure 6 shows a constrictor knot.[2] Inspection of the drawing yields the parameters shown in Fig. 6, from which we calculate

$$\det A = \det \begin{bmatrix} 1 & 0 & -\eta\epsilon^1 \\ -1 & 1 & -\eta\epsilon^0 \\ 0 & -\epsilon^1 & 1 - \eta\epsilon^1 \end{bmatrix} = 1 - \eta\epsilon(2 + \epsilon), \qquad (28)$$

so that the hold-fast condition can be written

$$\eta \geqslant \eta_c = [\epsilon(2 + \epsilon)]^{-1}. \qquad (29)$$

For the geometry assumed in this paper, in which the diameter of the rope is much less than that of the pole, η will usually be small compared to 1. Thus comparison of (21) and (29) shows that the constrictor hitch will hold fast with even less friction than the ground-line hitch.

Note that if we cross out the first row and column of the

i	n_i	b_i	m_i
1	0	3	1
2	1	3	0
3	1	3	1

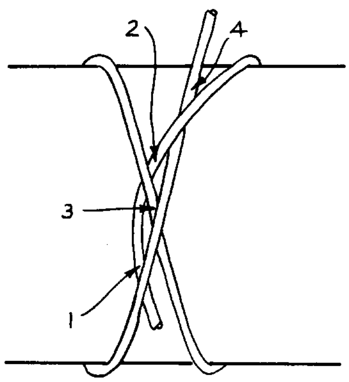


Fig. 6. Constrictor knot. The arrows indicate the beginnings of the four segments.

matrix A in (28), we get the 2×2 determinant

$$\det A' = \det \begin{bmatrix} 1 & -\eta \\ -\epsilon & 1 - \eta\epsilon \end{bmatrix} = 1 - 2\eta\epsilon, \quad (30)$$

which is the determinant of the A matrix for the hitch shown in Fig. 7. The hold-fast condition for this hitch is $\eta \geqslant 1/2\epsilon$. If this condition is satisfied, then the constrictor knot can hold fast even though $T_1 = 0$. Similarly, if we cross out the first two rows and columns of the matrix A in (28), we get the 1×1 determinant $1 - \eta\epsilon$, so that if $\eta \geqslant 1/\epsilon$ the constrictor hitch can hold fast even though and T_1 and $T_2 = 0$. Thus we can distinguish four friction regimes for the constrictor knot:

$$\text{(i) } \eta < [\epsilon(2+\epsilon)]^{-1},$$

hitch slips;

$$\text{(ii) } [\epsilon(2+\epsilon)]^{-1} \leqslant \eta < 1/2\epsilon,$$

hitch holds, with tension needed along the entire length of hitch;

$$\text{(iii) } 1/2\epsilon \leqslant \eta < 1/\epsilon,$$

hitch holds, but T_1 can be zero;

$$\text{(iv) } 1/\epsilon \leqslant \eta,$$

hitch holds, but T_1 and T_2 can be zero.

In general, by striking out successive rows and columns from the matrix A defined in (9), we get a sequence of hold-fast conditions, corresponding to increasing friction, in which the earlier parts of the hitch can be free of tension.

Comparison of Figs. 6 and 7 shows that the constrictor knot can be made by giving an extra tuck to the free end of the abbreviated constrictor knot. Since this tuck will be under a part of the rope near the end of the hitch, where the tension is high, it can bring about a large increase in the

i	n_i	b_i	m_i
1	1	2	0
2	1	2	1

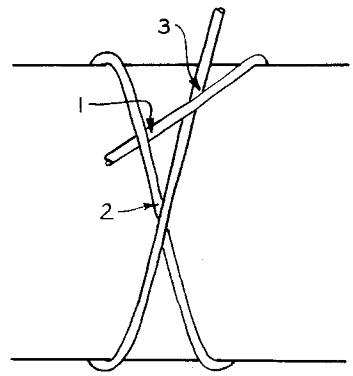


Fig. 7. Abbreviated constrictor knot. This knot is obtained from the constrictor knot by omitting the first segment.

holding power of the hitch. Figure 8 shows another attempt to improve abbreviated constrictor knot. In this case $\det A$ is calculated to be

$$\det A = \det \begin{bmatrix} 1 - \eta\epsilon & 0 & 0 \\ -\epsilon & 1 & -\eta \\ 0 & -\epsilon & 1 - \eta\epsilon \end{bmatrix} = (1 - \eta\epsilon)(1 - 2\eta\epsilon). \quad (31)$$

Suppose that $1 - 2\eta\epsilon > 0$, so that there is not enough friction to enable the abbreviated constrictor knot to hold-fast. Then $(1 - 2\eta\epsilon)(1 - \eta\epsilon)$ will also be positive, so that the extra loop added in going from Fig. 7 to Fig. 8 has brought us no improvement in the ability of the hitch to hold fast with little friction. This extra loop contributes nothing because T_1 can

i	n_i	b_i	m_i
1	1	1	1
2	1	3	0
3	1	3	1

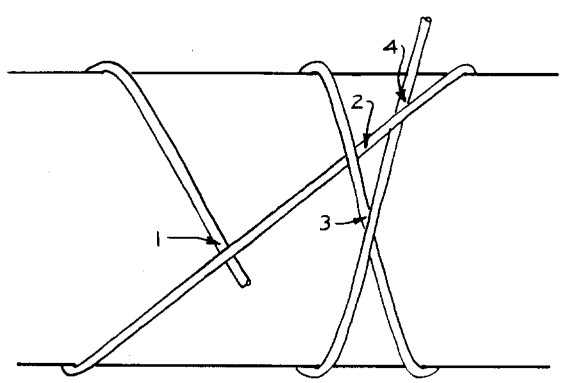
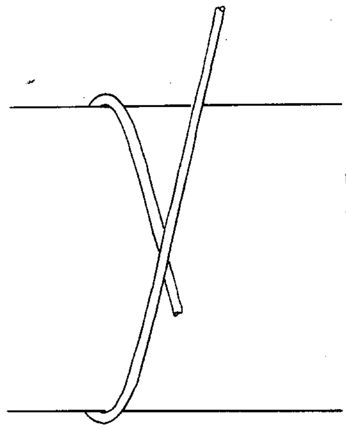


Fig. 8. Another attempt to improve the abbreviated constrictor knot. The arrows indicate the beginnings of the four segments.



i	n_i	b_i	m_i
1	1	1	1

Fig. 9. Hitch will slip if the upper segment is lifted away from the pole.

only be nonzero if $\eta\epsilon \geq 1$ and we are considering a $\eta\epsilon < \frac{1}{2}$ situation. Thus we gain no holding power by adding a feebler disconnected loop. However, we gain considerable holding power by tucking the free end of the abbreviated constrictor knot under a later segment of this hitch, as we do when we form the constrictor knot. It is clear from these examples that the most effective hitches are those that make most use of the high tension in the later stages of the hitch to produce large changes in the tensions of the segments that start beneath them.

VI. QUALIFICATIONS

We have assumed that when the last segment of a hitch is pulled the topology of the hitch remains unchanged, while the tension is transmitted along the hitch. Whether or not this happens depends upon the topology of the hitch. For example, the criteria of Sec. IV applied to the hitch shown in Fig. 9 gives the hold fast condition as $\eta\epsilon \geq 1$, the same as for a clove hitch. However, irrespective of the values of η and ϵ, the hitch in Fig. 9 would slip if the tight end of the rope were lifted out of the plane of the paper, whereas the clove hitch would continue to hold. Thus when we are comparing hitches, we must supplement comparisons of η_c by comparison of the abilities of the hitches to withstand pulls in various directions. The topological integrity of any hitch is improved if it is under tension, and instructions for tying these hitches usually suggest that the rope in each hitch be pulled tight before the full load is applied.

Fig. 10. Effective value of η will be smaller in (a) than in (b), since the two nearby cords in (a) share the downward pressure of the upper rope.

We have also assumed that a rope suffers no change in tension when it crosses *over* another turn, but only when it is squeezed between an upper turn and the pole. If we relax this assumption, we have to modify our definition of a segment to allow segments to start either when the rope passes under another turn or when it passes over another turn. When the rope passes over another turn, its change in tension will be governed by a relation such as (1), with θ a small angle if the diameter of the rope is small compared with the diameter of the pole. Its change in tension will not depend upon the tension in the rope it crosses over. We would also have to modify (2) to take account of the fact that the tension T in the upper cord is changing as it crosses over the lower cord.

We have not included in our discussion hitches in which the rope makes a turn or half-turn about itself, such as happens at the top of a timber hitch (Fig. 195 of Ref. 2). If the friction of rope on rope is small, this contact of the rope with itself can be ignored. Otherwise it can be treated by means of a multiplicative factor, as in (1).

It should be noted that we have assumed that the same constant η is used in (2) for all crossings. However, it is clear from Fig. 10 that the effective value of η may be modified by the proximity of several lower ropes which share the downward pressure of the upper rope.

It is evident that an accurate treatment of all aspects of a hitch would be a very complicated task. However, we believe that the somewhat simplified model discussed in the earlier sections of this paper illustrates the interplay between friction and topology in determining the ability of a hitch to form a self-locking unit that can withstand any pull.

[1]A. S. Ramsey, *Statics* (Cambridge U. P., Cambridge, England, 1969), p. 232.
[2]C. W. Ashley, *The Ashley Book of Knots* (Doubleday, Garden City, NY, 1944).